Asymptotically Safe cosmology: An effective description

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based on

M. Hindmarsh and I. D. Saltas, *PRD 86 (2012) 064029*, arXiv: 1203.3957 [gr–qc]

BritGrav 2013, Sheffield

Overview of the talk

- Key points :

  - The emerging cosmology of the **Asymptotic Safety** scenario for quantum gravity in the approximation of the Einstein–Hilbert truncation.

  - Understanding the quantum corrections in the RG-improved action from an **effective point of view**, by means of an $f(R)$ model.

- Emerging cosmology :

  - Primordial inflation: de Sitter solution(s), primordial spectra.

  - Late time cosmological evolution: Radiation/Matter evolution, late time de Sitter attractor.

  - Evasion of solar system tests.

Asymptotic Safety

- Weinberg' s asymptotic safety scenario:[1] Gravity is fundamental and **non-perturbatively** quantisable, provided a UV fixed point exists under the RG flow: Avoids UV divergences and theory "safe" in the UV.

- *The scale dependent effective average action* of gravity at energy scale k, Γ_k, satisfies an 1-loop type, **exact functional renormalisation group equation**[2]

$$k\partial_k\Gamma_k[\phi] = \frac{1}{2}\,Tr\left(\frac{\delta^2\Gamma}{\delta\phi\delta\phi} + R_k\right)^{-1} k\partial_k R_k$$

- Solution yields a *family of effective actions* Γ_k smoothly connected from UV ($k \longrightarrow \infty$) to IR ($k \longrightarrow 0$), through the system of beta functions for the couplings c_i,

$$k\partial_k c_i(k) = \beta_i(c_1, c_2, \ldots c_j)$$

- Effective action

$$\Gamma_k[g] = \int d^4x\sqrt{g}\; f(g_{\mu\nu}, R^{\mu}{}_{\nu\kappa\lambda})$$

- The *lowest order* approximation is the Einstein–Hilbert truncation:

$$\Gamma_k = \int d^4x\sqrt{-g}\,\frac{R - 2\Lambda(k)}{16\pi G(k)}$$

An **non–trivial (interacting) UV fixed point exists** for the dimensionless couplings $(g, \lambda) \simeq (0.016, 0.25)$, and a "free" Gaussian Fixed Point (GFP) at $(g, \lambda) = (0, 0)$. [3]

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[1] S. Weinberg in General Relativity, an Einstein Centenary Survey (1979)

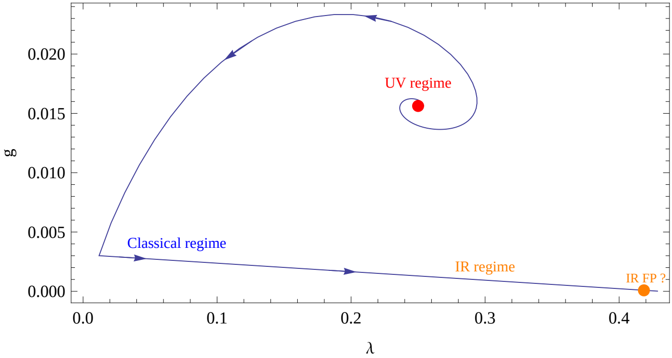
[2] C. Wetterich Phys. Lett. **B 301**, 90 (1993).

[3] M. Reuter, Phys. Rev. D 57 971 (1998).

The phase diagram of the Einstein–Hilbert truncation

$$\frac{\partial \lambda}{\partial \ln k} = \beta_\lambda(g,\lambda) \equiv -2\lambda - 2g - \frac{24g\left(3g+\frac{1}{2}(1-3\lambda)\right)}{2g-\frac{1}{2}(1-2\lambda)^2} \qquad \lambda \equiv \frac{\Lambda(k)}{k^2}$$

$$\frac{\partial g}{\partial \ln k} = \beta_g(g,\lambda) \equiv 2g + \frac{24g^2}{4g-(1-2\lambda)^2} \quad {}^{4} \qquad g \equiv 24\pi G(k) \times k^2$$

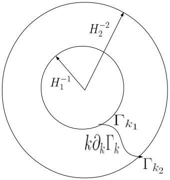


- Possible strong IR effects at $\lambda \sim \mathcal{O}(1)$?

[4] Beta functions are calculated in the optimised cut-off of Ref. D. F. Litim, Phys. Lett. B 486 92 (2000).

An effective description of the RG improved action

- The goal: To associate the infrared RG cut–off k in $G_k = G(k), \Lambda_k = \Lambda(k)$, with a physical scale in cosmology and implement the RG information (consistently) into the cosmological equations.



- A heuristic ansatz: $k = \rho R$.

Scalar curvature as a *natural IR cut–off* in an expanding universe:
A coarse-grained view of the gravitational force.

- Compare with RG improvement of the effective potential in scalar field theories at 1–loop [5].

$$V_k[\phi] = \frac{1}{4!}c(k)\phi^4 \quad \xrightarrow{k=\rho\phi} \quad V[\phi] = \frac{1}{4!}\frac{c_0}{1-b(c_0)\ln(\rho\phi/k_0)}$$

- The Einstein–Hilbert action can be thought as the lowest order term in a derivative expansion for the metric.

$$\mathcal{L}_k[g] = \frac{R-2\Lambda(k)}{16\pi G(k)} \quad \xrightarrow{k^2=\rho R} \quad \mathcal{L}[g] = \rho R^2\left(\frac{1-2\rho\lambda(R)}{192\pi^2 g(R)}\right)$$

- Quantum corrections of the RG improved action, included in the running of $G(k)$, $\Lambda(k)$, are now "absorbed" in a non-linear, effective $f(R)$ model.

- On the UV fixed point, $f(R)$ model behaves as R^2 gravity (renormalisable). Scalaron mass also vanishes there, reflecting the absence of fundamental scale.

- Dimensionless parameter $\rho \equiv k^2/R$ controls how curvature "follows" the cut-off k. Cosmological considerations will reveal its value.

[5] S. R. Coleman & E. J. Weinberg, Phys. Rev. D 7, 1888, (1973)

Background cosmological dynamics: From Planck scale to late time expansion

- Effective RG improved $f(R)$ model is defined along the RG flow as

$$g(R), \lambda(R) \longrightarrow f[R, g(R), \lambda(R)]$$

$$R\frac{\partial g}{\partial R} = \beta_g(g,\lambda), \qquad R\frac{\partial \lambda}{\partial R} = \beta_\lambda(g,\lambda)$$

- Background cosmological evolution can be studied by **RG-improving** the dynamics to implement the information from the beta functions.

$$x_1' = -1 - x_3 - 3x_2 + x_1^2 - x_1 x_3 + x_4$$

$$x_2' = \frac{x_1}{x_3} - x_2(2x_3 - x_1 - 4)$$

$$x_3' = \frac{-x_1 x_3}{m(r)} - 2x_3(x_3 - 2)$$

$$x_4' = -2x_3 x_4 + x_1 x_4$$

$$g' = \frac{\beta_g}{2R}\left(\frac{f^2}{f_R^2 R - f_R f - f_{RR} f R}\right)\left(\frac{x_3' x_2 - x_2' x_3}{x_2^2}\right)$$

$$\lambda' = \frac{\beta_\lambda}{2R}\left(\frac{f^2}{f_R^2 R - f_R f - f_{RR} f R}\right)\left(\frac{x_3' x_2 - x_2' x_3}{x_2^2}\right)$$

$$\Omega_m \equiv \frac{\kappa^2 \rho_m}{3H^2 f_R} = 1 - x_1 - x_2 - x_3 - x_4$$

Dimensionless dynamical variables

$$x_1 = \frac{-\dot{f}_R}{H f_R}, \quad x_2 = \frac{-f}{6H^2 f_R}$$

$$x_3 = \frac{R}{6H^2}, \quad x_4 = \frac{\kappa^2 \rho_r}{3H^2 f_R}$$

$f(R)$ model input enters through the function $m = m(r)$.

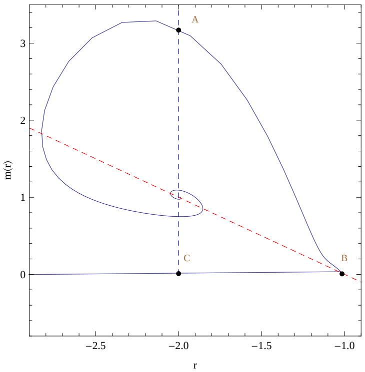
$$m(R) \equiv \frac{R f_{RR}}{f_R}$$

$$r(R) \equiv -\frac{R f_R}{f} = \frac{x_3}{x_2}$$

Remember : $f \equiv f(R, g(R), \lambda(R))$

- Function $m = m(r)$ determines form of $f(R)$ and needed to close the system!

- The behavior of the function $m = m(r)$ is able to characterize the asymptotic behavior of the $f(R)$ model: *fixed points and their stability*.



$$f \equiv f\Big(R, g(R), \lambda(R)\Big)$$

$$m(R) \equiv \frac{Rf_{RR}}{f_R}, \qquad r(R) \equiv -\frac{Rf_R}{f}$$

de Sitter points: A (unstable), C (stable)

Stability : $0 < m(r)|_{r=-2} < 1$.

Matter point: B (unstable spiral)

Stability : $m(r)|_{r=-1} \simeq +0, \ \frac{dm(r)}{dr}\Big|_{r=-1} > -1$

- In the presence of radiation, i.e $x_4 \neq 0$, a radiation point exists in the vicinity of the matter one. [6]
- Numerical investigation shows that viable background evolution requires that

  $0.9 \lesssim \rho \equiv \frac{k^2}{R} \lesssim 1.1$, otherwise Universe does not evolve through matter domination and late time de Sitter.

[6] See Ref. L. Amendola, R. Gannouji, D. Polarski & S. Tsujikawa, Phys.Rev. D 75 083504 (2007).

Solar and astrophysical scales

- We expect solar and astrophysical scales to be recovered in the vicinity of the Gaussian Fixed Point $(g, \lambda) = (0, 0)$, away from strong UV or IR Renormalistion Group effects, e.g

$$R_{\rm sol}^{-1/2} \sim 1\mathrm{AU} \Rightarrow g_{\rm sol} \simeq R_{\rm sol} \times G_{\rm sol} \simeq 10^{-92}.$$

For small couplings the leading order contribution to the beta functions are

$$\frac{\partial \lambda}{\partial \ln R} \simeq -\lambda + \mathcal{O}(1) g,$$
$$\frac{\partial g}{\partial \ln R} \simeq g.$$

- Think of a perturbative expansion of the effective $f(R)$ model around a typical scale of interest $R = R_0$ and and match with the renormalisation conditions

$$f(R) = f(R_0) + f_R|_{R_0} (R - R_0) + \frac{1}{2} f_{RR}|_{R_0} (R - R_0)^2$$

$$\left.\frac{Rf_R - f}{2f_R}\right|_{R_0} = \Lambda_0 \qquad f_R|_{R_0} = \frac{\tilde{\kappa}^2}{8\pi G_0}$$

$$f(R) \simeq \frac{\tilde{\kappa}^2}{G_0}(R - 2\Lambda_0) + 6(2-\rho)\rho(R - R_0)^2$$

- The scalaron mass in the GFP linear regime ($g \sim \lambda \ll 1$) is of the *Planck order*

$$m^2_{\rm eff,0}(g, \lambda) \simeq \frac{1}{36(2-\rho)} \frac{R_0}{g} \simeq \frac{1}{36(2-\rho)} \frac{\tilde{\kappa}^2}{8\pi G_0}$$

Heavy scalaron mass prevents from observable deviations from GR.

- *Positivity of scalaron's mass* at solar scales requires that

$$\frac{k^2}{R} \equiv \rho < 2$$

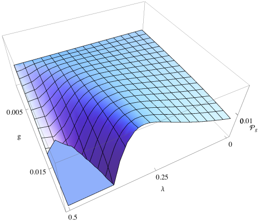
The inflationary power spectra

Einstein frame action $\widetilde{S} = \int d^4x\sqrt{-\tilde{g}}\left(\frac{\widetilde{R}}{16\pi G_0} - \frac{1}{2}(\nabla\Phi)^2 - U(\Phi)\right)$

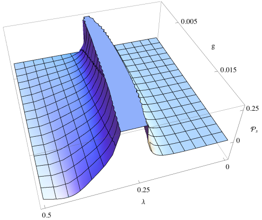
$\Phi = \Phi(g,\lambda)$ $\mathcal{P}_s\left(\Phi(g,\lambda)\right) = \frac{128\pi}{3}\left.\frac{U^3}{m_p^2 U_\Phi^2}\right|_{k=aH}$ $\mathcal{P}_g(\Phi(g,\lambda)) = \frac{128}{3}\left.\frac{U}{m_p^4}\right|_{k=aH}$

- Primordial inflation is expected at high energies, i.e in the vicinity of the UV RG fixed point, where $(g\lambda)_{UV} = (G\Lambda)_{UV} \sim 10^{-2}$, implying that $P_g \sim 10^{-2}$.
- Scalar fluctuations are of same order with gravitational ones [7], $P_s \sim 10^{-2}$.
- Primordial fluctuations are too large to agree with observations. Possible solutions?

$\mathcal{P}_g\,(g,\lambda)$



$\mathcal{P}_s(g,\lambda)$



[7] More precisely, this value is derived assuming $\rho \equiv k^2/R \sim 1$, as required by late time evolution.

Conclusions

- **Asymptotic Safety** is a very promising approach for a UV completion of (metric) gravity beyond perturbation theory. Recent developments strongly suggest the existence of a UV fixed point associated with a finite number of relevant directions, for truncations of higher order.

- **Quantum fluctuations** play an important role in cosmology especially at early times: The running of the gravitational couplings with energy can provide an explanation for the small value of the cosmological constant, as well as provide a unified framework for both the early and late time acceleration of the universe.

- To reveal the cosmological dynamics of the action we implemented the quantum corrections into an **effective $f(R)$ model** through a heuristic cut–off identification: Scalar curvature playing the role of an infrared cut–off in an expanding universe, $k^2 \sim \mathcal{O}(1)R$.

- The **emerging cosmology** suggests that the universe starts from an unstable de Sitter state, and evolves towards a radiation/matter evolution, attracted to a de Sitter phase in the IR with a small cosmological constant. Leading behavior in the UV fixed point regime is that of R^2 gravity. Scalaron effectively decouples at solar scales.

- **Primordial inflation** gives large fluctuations, $(G\Lambda)_{UV} \sim \mathcal{O}(10^{-2})$.

- **Open questions & work in progress**: How generic are the features found? Can a viable inflationary period emerge in a natural way? Higher truncations?