Generating inter-galactic magnetic fields in the early universe

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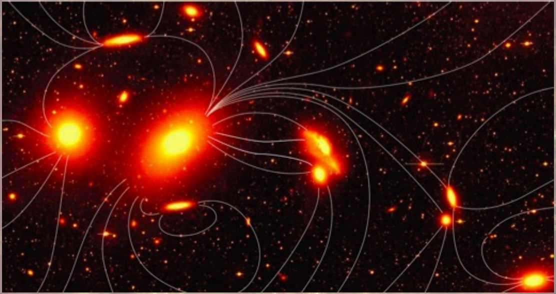
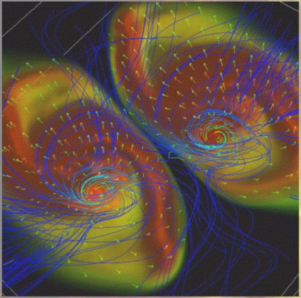
Observed inter-galactic magnetic fields



Artists impression of the intergalactic magnetic field in the Virgo Cluster (Science, Vol 311, P788).

- Halos around gamma-ray images Ando & Kusenko 2010
- TeV blazers Neronov & Vovk 2010 & (using Fermi/LAT) Tavecchino et al. 2010
- Magnitude: $10^{-17} - 10^{-14}$G.
- Amplified from initial seed fields.

Amplification



Magnetic filed lines and velocities in a magnetised cloud. Simulation by Matsumoto 2004.

Amplification of seed field

- Dynamo Mechanism transfers energy from kinetic into magnetic.
- Needs a seed field of size $10^{-30} - 10^{-12}$G.
- Adiabatic compression of magnetised cloud transfers energy from gravitational potential into magnetic.
- Needs a seed field of at least 10^{-20}G.

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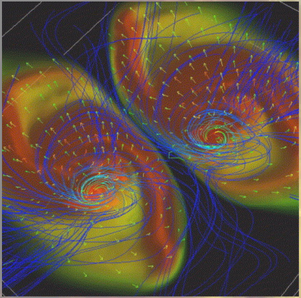

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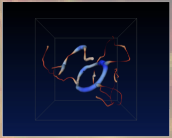
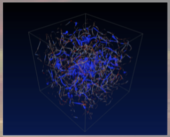
- The initial seed fields are likely to be cosmological in origin.

Where does the seed field come from?

Options

- Inflation
- Phase transitions
- Preheating or reheating
- Exotic Physics
- Vorticity & Velcoity differences





Time evolution of vortex core lines. Color and line thickness represent the vorticity (blue: high vorticity values).

http://www.zib.de/en/numerical-methods.html

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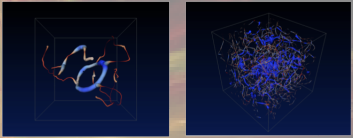

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Naturally ocurring vorticity

- Vorticity is generated in the early universe at 2nd order.
  Christopherson, Malik & Matravers 2009
- Combined with a velocity difference between charged species this can generate a magnetic field.
- We are looking for the highest order source term and its effect on the magnitude of the magnetic field.

Using perturbations

- We use perturbative methods, following Bardeen's formalism.
- We expand around a FRW background up to 2nd order and work in flat gauge.

Perturbed metric terms

$$\begin{aligned} g_{00} &= -a^2\left[1 + 2\phi_1 + \phi_2\right] \\ g_{0i} &= a^2\left[B_{1i} + \frac{1}{2}B_{2i}\right] \\ g_{ij} &= a^2\left[\delta_{ij} + 2C_{1ij} + C_{2ij}\right] \end{aligned}$$

- We also expand our matter components and fields to 3rd order.

Perturbed components

$$\rho(t, x^i) = \rho_0(t) + \delta\rho_1(t, x^i) + \frac{1}{2}\delta\rho_2(t, x^i) + \frac{1}{6}\delta\rho_3(t, x^i)...$$

Maxwell's Equations

Covariant Maxwell Equations

Constraint Equations

$$\mathcal{E}^{\mu}{}_{,\mu} + \Gamma^{\mu}_{k\mu}\mathcal{E}^{k} - \dot{u}_{\mu}\mathcal{E}^{\mu} = \hat{\rho} - 2\omega^{\mu}\mathcal{M}_{\mu}$$

$$\mathcal{M}^{\mu}{}_{,\mu} + \Gamma^{\mu}_{k\mu}\mathcal{M}^{k} - \dot{u}_{\mu}\mathcal{M}^{\mu} = -\omega^{\mu}\mathcal{E}_{\mu}$$

Evolution Equations

$$\dot{\mathcal{E}}^{\lambda\perp} = (\omega^{\lambda}_{\nu} + \sigma^{\lambda}_{\nu} - \frac{2}{3}\theta h^{\lambda}_{\nu})\mathcal{E}^{\nu} + \epsilon^{\lambda\nu\mu}\dot{u}_{\nu}\mathcal{M}_{\mu} - \epsilon^{\lambda\nu\mu}(\mathcal{M}_{\nu,\mu} - \Gamma^{k}_{\nu\mu}\mathcal{M}_{k}) - \mathcal{J}^{\lambda}$$

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Current equations

$$\mathcal{J}^{\mu} = \hat{\sigma}E^{\mu}$$

$$\mathcal{J}^{j}_{(s)} = eZ_{(s)}\hat{n}_{(s)}v^{j}_{(s)}$$

Introducing matter

We consider a 3 fluid system:
protons (p), electrons (e), and radiation (γ).

Governing Equations

Momentum conservation

$$\dot{V_\alpha} + \left[\frac{Q_\alpha}{\rho_\alpha + P_\alpha}(1 + c_\alpha^2) - 3Hc_\alpha^2\right]V_\alpha + \phi + \frac{1}{\rho_\alpha + P_\alpha}\left[\delta P_{1\alpha} + \frac{2}{3}\frac{\nabla^2}{a^2}\Pi_\alpha + Q_\alpha V - \sum_\beta f_{\alpha\beta}\right] = 0$$

Interactions between species

$$f_{\alpha\beta} = \alpha_{\alpha\beta}(V_\alpha - V_\beta)$$

$$\alpha_{pe} = \frac{n^2e^2}{4\pi\epsilon_0\sigma_C}, \quad \alpha_{e\gamma} = \frac{4}{3}nc\sigma_T\rho_\gamma, \quad \alpha_{p\gamma} = \frac{4\beta^2}{3}nc\sigma_T\rho_\gamma$$

Number density of particles

$$n_e = n_p = \frac{2\zeta(3)\eta_{B0}}{\pi^2}T^3$$

Initial results

Our assumptions:

- No tensors.
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We combine and expand equations to get full 3rd order evolution and constraint equations using Cadabra http://cadabra.phi-sci.com/ and we find a source term at 3rd order.

Source term for magnetic fields

$$\mathcal{M}_3{}^i = \frac{3e\hat{n}_0 k^i}{a\hat{\sigma}k^2}\ \omega_2{}^j\ Dv_{(pe)1j}$$

$$\omega'_{ij} - \mathcal{H}\omega_{ij} = \frac{9a}{8\rho_0^2}\delta\rho_{,[j}\delta P_{,i]} = S_{ij}\ , \qquad Dv_{(pe)1j} = \partial_j(v_{1(p)} - v_{1(e)})$$

What's next?

- Comparison of the magnitude and scale of the magnetic fields produced to those needed in amplification mechanisms.

Full power spectrum calculation

$$\langle \mathcal{M}^*(\mathbf{k}_1,\eta)\mathcal{M}(\mathbf{k}_2,\eta)\rangle = \frac{2\pi}{k^3}\delta(\mathbf{k}_1-\mathbf{k}_2)\mathcal{P}_{\mathcal{M}}(k,\eta)$$

$$\begin{aligned}\langle {\mathcal{M}_3}^*(\mathbf{k}_1,\eta)\mathcal{M}_3(\mathbf{k}_2,\eta)\rangle \quad = \quad & \frac{9}{4k_1k_2(2\pi)^3}\left(\frac{e\hat{n}_0\eta}{a\hat{\sigma}}\right)^2\int_{\eta_0}^{\eta}\tilde{\eta_1}^{-1}d\tilde{\eta_1}\int_{\eta_0}^{\eta}\tilde{\eta_2}^{-1}d\tilde{\eta_2}\int d^3\tilde{\mathbf{k}_1}\int d^3\tilde{\mathbf{k}_2} \\ & \langle S^*(\tilde{\mathbf{k}_1},\tilde{\eta_1})Dv^*_{(pe)1}(\mathbf{k}_1-\tilde{\mathbf{k}_1},\eta)S(\tilde{\mathbf{k}_2},\tilde{\eta_2})Dv_{(pe)1}(\mathbf{k}_2-\tilde{\mathbf{k}_2},\eta)\rangle\end{aligned}$$

Extensions:

- Include tensors in our calculations.
- Use a full Boltzmann calculation.

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Thanks for listening!