The cosmic-no-hair conjecture for 'almost-Bianchi' space-times

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Evolution of the universe on large scales

Observations and models

On large scale our universe is almost spatially homogeneous. Hence the large scale evolution is studied by approximating the universe by a Bianchi spacetime.

The universe undergoes an accelerated expansion, suggesting a posititve cosmological constant $\Lambda > 0$.

Questions arising

1. What is the long-term evolution of an exact Bianchi space-time?
2. Are these predictions sensitive to perturbations, i.e. does the presence of small inhomogeneities alter the long term evolution?

Cosmic no-hair theorem for exact Bianchi space-times

Wald 1983

Suppose we are given a spatially homogeneous (Bianchi) space-time

- with ${}^{(3)}R \leq 0$ (not Bianchi IX),
- that is initially expanding,
- satisfies $G_{\mu\nu} + \Lambda g_{\mu\nu} = T_{\mu\nu}$ with $\Lambda > 0$,
- where $T_{\mu\nu}$ satisfies the dominant and strong energy conditions

Then this Bianchi space-time evolves exponentially towards de Sitter space.

Due to the continued expansion, we observe that at late times

- spatial geometry and matter distribution are smoothed out,
- universe homogenises and isotropises,
- universe looks locally more and more like de Sitter space,
- no distinguishable features $\rightarrow$ no hair.

Question

Is the same true for almost spatially homogeneous space-times?

Related non-linear stability results

Theorem (L. & Valiente Kroon 2013)

Suppose we are given Cauchy initial data for the Einstein-Euler system with $\Lambda > 0$ *and equation of state* $p = \frac{1}{3}\mu$*. If the initial data is sufficiently close to data for a FLRW cosmological model with* $p = \frac{1}{3}\mu$*, same* Λ *and spatial curvature* $k = 1$*, then*

- *the development exists globally towards the future,*
- *is future geodesically complete,*
- *remains close to the FLRW solution.*

- A similar result exists for the stability of de Sitter in the class of Einstein-Maxwell space-times [Friedrich 1991, L. & Valiente Kroon 2012].

Comments on these results

- The stability result is fully non-linear.
- Small perturbations are assumed.
- The result makes use of the conformal Einstein field equations (CEFE), originally due to Friedrich, adapted for radiation fluids (Einstein-Maxwell).
- The stability result can be extended as long as the reference space-time (background) is shown to be a regular solution of the CEFE.
- Alternative proof of stability of FLRW with an irrotational perfect fluid with $1 \leq \gamma \leq 4/3$ given by Rodnianski & Speck 2012

Detailed decay rates for exact Bianchi

We consider Einstein-Maxwell and Einstein-Euler systems with $\gamma = \frac{4}{3}$.
Using a $3+1$-orthonormal frame approach one get can more precise decay rates.
Here $\lambda = \sqrt{\Lambda/3}$

$$
\begin{aligned}
\lambda < H &< \lambda \coth(\lambda t) \\
0 < C_1 e^{\lambda t} \leq L &\leq C_2 e^{\lambda t} \\
n_{ab}, a_a &= O(L^{-1}) \\
\sigma_{ab},\ {}^{(*)}S_{ab},\ {}^{(3)}R &= O(L^{-2}) \\
\mu, p, q_a, \pi_{ab} &= O(L^{-4}) \\
E_a, B_a &= O(L^{-2}) \\
E_{ab}, H_{ab} &= O(L^{-2})
\end{aligned}
$$

Regular solution to the CEFE and stability

- Rescale the metric $\tilde{g}_{\mu\nu} = \Theta^2 g_{\mu\nu}$ with $\Theta = L^{-1}$
- Define conformal time $\tau = \int_0^t \frac{1}{L(s)} \mathrm{d}s \Rightarrow \tau$ is finite at conformal infinity
- Use $\tilde{g}$-orthonormal frame $\Rightarrow$ rescaled quantities are finite at conformal infinity.
- The Bianchi space-times considered here give regular solution to the CEFE.
- Use them as reference space-times for the stability theorems.

Theorem

Given a small perturbation of a Bianchi space-time (except type IX) whose matter content is Einstein-Maxwell or a radiation fluid, then these 'almost Bianchi' space-times locally asymptote to de Sitter space at late times.

Cosmic no-hair for almost Bianchi space-times

- The theorem shows that Wald's result is stable.
- Almost Bianchi space-times (except Bianchi IX) satisfy the cosmic no-hair conjecture, i.e locally homogenise and isotropise to locally approach de Sitter at late times.
- The physical space-time has 'lost its hair'.
- However CEFE and conformal infinity retain the information of the 'approximate Bianchi-type' and matter model (hair style) in terms of non-vanishing rescaled quantities.

Discussion and outlook

- Results are local and make no statement about the global spatial topology.
- For Bianchi IX Wald shows that if Λ is sufficiently large they also satisfy cosmic no-hair conjecture.
- If ${}^{(3)}R > 0$ or $\Lambda = 0$ then recollapse is possible (Lin & Wald 1990)
- For other trace-free energy momentum tensor (null fluids, Vlasov, conformal scalar field) the regular CEFE are expected $\rightarrow$ generalisations seem possible. But at the moment no stability theorems using the CEFE are known.
- For general perfect flud case$(1 \leq \gamma \leq \frac{4}{3})$, Rodnianski & Speck 2012 should allow for proof of stability.

Thank you for listening