Critical phenomena at the threshold of immediate merger in binary black hole systems: the extreme mass ratio case

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BritGrav13, Sheffield 3/4 April 2013

Numerical relativity results

Pretorius & Khurana 2007, Sperhake et al 2009:

- Motivated by collider physics
- Equal mass collisions with large(ish) boost, $k \sim$ a few
- Fine-tuning to critical impact parameter b_* gives several orbits
- $\Delta E/E \sim 0.01 \ldots 0.1$ per orbit for a few orbits
- Conjecture: with sufficient fine-tuning, almost all the energy can be radiated
- Toy model: zoom-whirl orbits around Schwarzschild

$$\text{number of orbits} \simeq -\frac{1}{2\pi} \ln |b - b_*|$$

Our work: Add radiation reaction in the self-force approximation

Point particle under the effect of a (self-)force

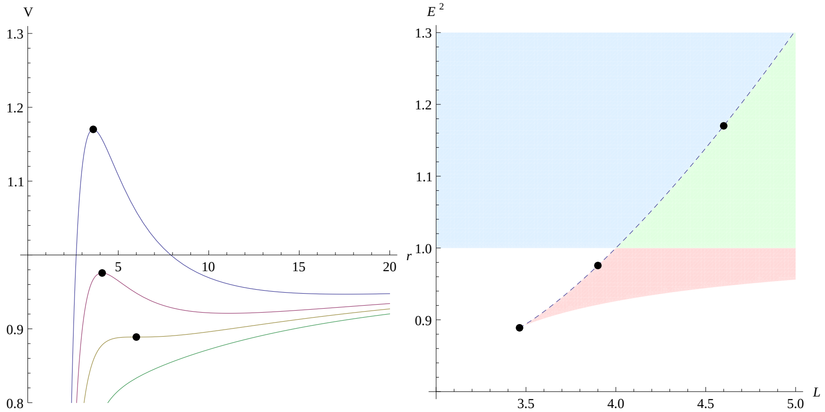
The normalisation $u^a u_a = -1$ becomes

$$E^2 = \dot{r}^2 + V, \qquad V(L,r) := \left(1 - \frac{2M}{r}\right)\left(1 + \frac{L^2}{r^2}\right)$$

and the equation of motion $u^a \nabla_a u^b = F^b$ becomes

$$\dot{E} = -F_t, \qquad \dot{L} = F_\varphi, \qquad \ddot{r} = -\frac{1}{2} V_{,rr} + F^r$$

Geodesics ($F^a = 0$)



$V(L, \infty) = 1$, so orbits with $E > 1$ are unbound

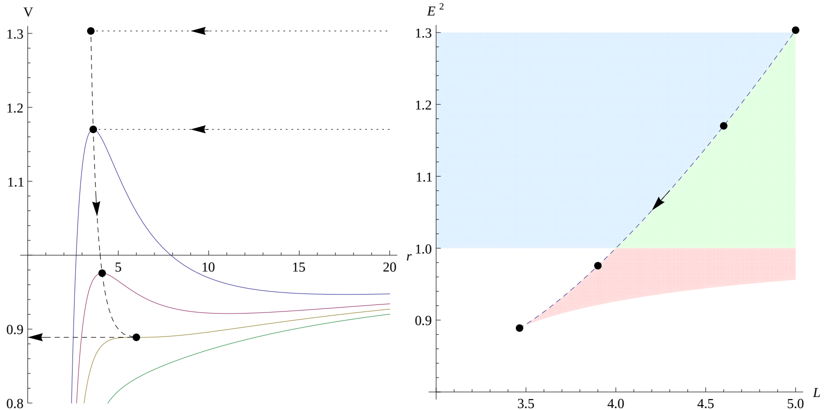
What is the critical solution?

Teleological definition: the critical solution sits on the ridge between plunge and scatter.
Local definition: the critical solution is as circular as radiation reaction allows.
Practical implementation: With $\eta \ll 1$ the mass ratio, make the "slow time" ansatz

$$r_{\rm crit}(\tau) = r_0(\hat{\tau}) + O(\eta), \qquad \hat{\tau} := \eta\tau$$

and similarly for E, L, φ.

Critical solution with radiation reaction



Input from self-force calculations

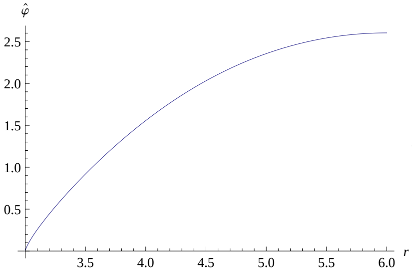
Numerical calculation of self-force on unstable circular geodesics of Schwarzschild, for $3.02M \leq r \leq 6M$. Extrapolate to light ring:

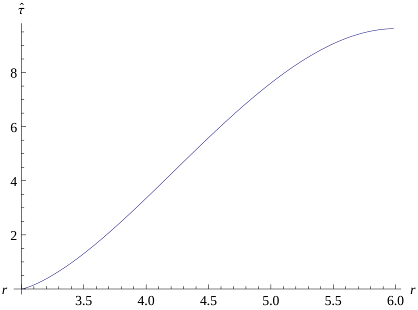
$$\dot{E} \sim \eta(r - 3M)^{-\alpha}$$

Best fit $\alpha \simeq 1.77$.

Plots of the critical solution

Integrate equation of motion with self-force to obtain slow angle and time $\hat{\varphi}(r)$ and $\hat{\tau}(r)$:





Angle and time are finite at $E = \infty \Leftrightarrow r = 3M$.

Perturbations of the critical solution

With gauge choice $\delta L = 0$,

$$\ddot{\delta r} \simeq -\frac{1}{2} V_{,rr}(\hat{\tau})\, \delta r$$

Use approximations

- Energy conservation during approach
- WKB during quasicircular phase

to calculate

$$E_f = E_f(\eta, E_i, b)$$

Critical scaling results

Simplified form for $E \gg 1$ is

$$\varphi_f - \varphi_i \simeq -\ln|b - b_*|$$

and

$$E_f^{-\gamma} - E_i^{-\gamma} \simeq -\eta \ln|b - b_*|$$

where best fit $\gamma := 2(\alpha - 1) \simeq 1.54$

$E_f \ll E_i$ requires only limited fine-tuning as $E_i \to \infty$.

Discussion

Results:

- New type "Ia" of critical solution
- Critical solution spirals *out* from $r = 3M$ to $r = 6M$
- From infinite energy to plunge in finite time and angle

Open questions:

- Blowup of self-force at the light ring
- Self-force beyond the adiabatic regime
- Comparable mass binaries
  - There is only one critical solution (for given mass ratio)
  - $b_* \sim r_{\rm crit} \sim E_{CM} \sim \sqrt{k}$: critical solution spirals *in*