The good, the bad and the ugly: including matter loops in the graviton two-point function in de Sitter

Markus B. Fröb

Universitat de Barcelona, Spain

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work done with and under the supervision of E. Verdaguer and A. Roura, based on JCAP 08 (2012) 009 (arXiv:1205.3097)

Introduction

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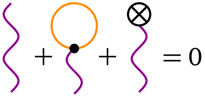
- Semi-classical background: 
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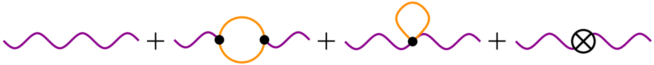

Milestones of the calculation

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- Starting point: $S = \frac{1}{\kappa^2} \int (R[g+\kappa h] - 2\Lambda) \underbrace{\sqrt{-(g+\kappa h)}\, \mathrm{d}^n x}_{\mathrm{d}x} + S_\mathrm{M}[g+\kappa h]$

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  $S_\mathrm{CS}[g,\phi] = -\frac{1}{2} \int \left(\nabla^a \phi \nabla_a \phi + \frac{(n-2)}{4(n-1)} R \phi^2 \right) \mathrm{d}x$

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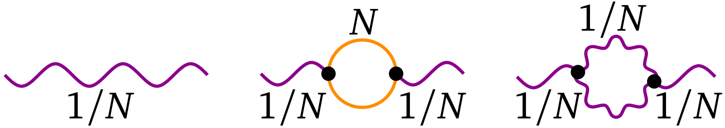
- 

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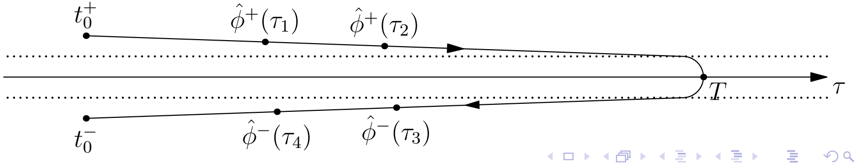
- Alternatively: closed-time-path (CTP)



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- explicitly calculated in Poincaré patch of dS — Campos, Verdaguer '94,'96

Semiclassical background

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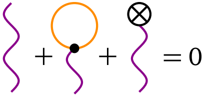

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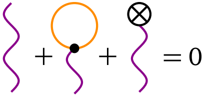

- Self-consistent de Sitter solution with $\Lambda_{\text{eff}} = \Lambda\left(1 + \#N\kappa^2\Lambda\right)$ (long known: for dS-invariant state, $\langle T_{ab}[\phi]\rangle_\phi \propto g_{ab}$)

Metric fluctuations

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$$\mathcal{G}^{AB}_{abcd}(x,y) = G^{AB}_{abcd}(x,y)$$
$$- \kappa^2 \int G^{AM}_{abmn}(x,x') \left(\frac{\delta^2 S^{(2)}_{\text{eff}}[h^\pm]}{\delta h^M_{mn}(x') h^N_{pq}(y')} \right) G^{NB}_{pqcd}(y',y) \,\mathrm{d}x' \,\mathrm{d}y' + \mathcal{O}\left(\kappa^4\right)$$

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- Scalar, vector correlation function

$$\mathcal{G}^{AB}_{abcd}(x,y) \propto \frac{1}{\triangle} \left(\frac{\delta^2 S^{(2)}_{\mathrm{eff}}[h^\pm]}{\delta h^A_{ab}(x')h^B_{cd}(y')} \right)$$

- Full result (the Ugly)

$$\mathcal{G}^{-+}_{abcd}(\eta,\eta',\mathbf{p}) = -\mathrm{i}\left(P_{ac}P_{bd}+P_{ad}P_{bc}-P_{ab}P_{cd}\right)\left[\frac{(|\mathbf{p}|\eta-\mathrm{i})(|\mathbf{p}|\eta'+\mathrm{i})}{2H^2\eta^2(\eta')^2|\mathbf{p}|^3}\mathrm{e}^{-\mathrm{i}|\mathbf{p}|(\eta-\eta')}\left(1+6\alpha\kappa^2H^2\ln\left(\frac{\bar{\mu}}{H}\right)-(5\alpha-2\beta)\,\kappa^2H^2\right)\right.$$
$$+\frac{3}{2}\alpha\kappa^2\left(\frac{2}{|\mathbf{p}|\eta\eta'}\mathrm{e}^{-\mathrm{i}|\mathbf{p}|(\eta-\eta')}+S(\eta-\eta',\mathbf{p})\right)$$
$$\left.-\frac{3}{2}\alpha\kappa^2\frac{1}{\eta^2(\eta')^2}\left(I_2(\eta,\eta',\mathbf{p})+I_2^*(\eta',\eta,\mathbf{p})-I_3(\eta,\eta',\mathbf{p})+I_4(\eta,\eta',\mathbf{p})\right)\right]$$
$$+\frac{\mathrm{i}}{2}\alpha\kappa^2\left[12\delta^0_{(a}P_{b)(c}\delta^0_{d)}\frac{1}{\mathbf{p}^2}\left(\partial_\eta^2+\mathbf{p}^2\right)+9\delta^0_{(a}p_{b)}\delta^0_{(c}p_{d)}\frac{1}{(\mathbf{p}^2)^2}\partial_\eta^2+3\delta^0_{(a}p_{b)}\delta^0_{(c}p_{d)}\frac{\eta-\eta'}{\mathbf{p}^2}\partial_\eta\right.$$
$$-\delta^0_{(a}p_{b)}\delta^0_{(c}p_{d)}\eta\eta'-3\mathrm{i}\delta^0_a\delta^0_b\delta^0_{(c}p_{d)}\frac{\eta}{\mathbf{p}^2}\partial_\eta^2+3\mathrm{i}\delta^0_{(a}p_{b)}\delta^0_c\delta^0_d\frac{\eta'}{\mathbf{p}^2}\partial_\eta^2$$
$$\left.+\mathrm{i}\delta^0_{(a}\left(\delta^0_{b)}p_{(c}+p_{b)}\delta^0_{(c}\right)\delta^0_{d)}\eta\eta'\partial_\eta+\delta^0_a\delta^0_b\delta^0_c\delta^0_d\eta\eta'\partial_\eta^2\right]S(\eta-\eta',\mathbf{p}),$$

with

$$P_{ab}=\eta_{ab}+\delta^0_a\delta^0_b-\frac{p_ap_b}{\mathbf{p}^2},\quad S(\eta-\eta',\mathbf{p})=-\mathrm{i}\mathrm{e}^{-\mathrm{i}|\mathbf{p}|(\eta-\eta')}\mathcal{P}\frac{1}{\eta-\eta'}+\pi\delta(\eta-\eta'),$$

and

$$I_2(\eta,\eta',\mathbf{p})=|\mathbf{p}|^{-3}\mathrm{e}^{\mathrm{i}|\mathbf{p}|(\eta+\eta')}\left(|\mathbf{p}|\eta+\mathrm{i}\right)\left(|\mathbf{p}|\eta'+\mathrm{i}\right)\left[\mathrm{Ein}\left(-2\mathrm{i}|\mathbf{p}|\eta\right)+\ln\left(2\mathrm{i}|\mathbf{p}|\eta\right)+\gamma\right]$$
$$+|\mathbf{p}|^{-3}\mathrm{e}^{-\mathrm{i}|\mathbf{p}|(\eta-\eta')}\left(|\mathbf{p}|\eta-\mathrm{i}\right)\left(|\mathbf{p}|\eta'+\mathrm{i}\right)\ln\left(-2|\mathbf{p}|\eta\right)$$
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$$I_4(\eta,\eta',\mathbf{p})=|\mathbf{p}|^{-3}\mathrm{e}^{\mathrm{i}|\mathbf{p}|(\eta-\eta')}\left(|\mathbf{p}|\eta+\mathrm{i}\right)\left(|\mathbf{p}|\eta'-\mathrm{i}\right)\left[\mathrm{Ein}\left(-2\mathrm{i}|\mathbf{p}|(\eta-\eta')\right)+\ln\left(2\mathrm{i}|\mathbf{p}|(\eta-\eta')\right)+\gamma\right].$$

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- super-horizon modes: $\delta^2 \rightarrow \frac{\kappa^2 H^2}{16\pi^3}(1 + \kappa^2 H^2 \text{const.}) + \mathcal{O}\left(|\mathbf{p}|\eta\right)$ (no logarithmic running $\sim \ln|\mathbf{p}|$, invariant under subset of dS isometries) Weinberg '05,'10, Chaicherdsakul '06, Adshead, Easther, Lim '09

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- Corrections much too small to be observable (the Bad) even by Planck 1303.5075, 1303.5076

Conclusions and Outlook

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- Generalization to other types of matter (main ingredient: stress tensor two-point function)

Thank you for your attention!

Questions?