Warped AdS_3 Black Holes: are they classically stable?

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Outline

Warped AdS_3 black holes

Set of complaints/interesting features:

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- (2+1)-dimensional!
- **Not** solutions of Einstein gravity!
- **Massive** graviton!
- **No** static observers outside the black hole!
- **No** immediate clear notion of time!

Warped AdS_3 black holes

WHAT?

WHAT?

... are they at least classically stable?

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- It retains much of the **conceptual complexity** of 4D gravity.
- Einsten gravity equivalent to a **Chern-Simons gauge theory**.

Achucarro, Townsend (1986), Witten (1988,1989)

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with:

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- **Massive** propagating degree of freedom!...
- **Third-order** derivative theory...

Metric of warped AdS_3 black holes

Spacelike stretched black hole:

$$ds^2 = dt^2 + \frac{\ell^2 dr^2}{4R^2(r)N^2(r)} + 2R^2(r)N^\theta(r)dtd\theta + R^2(r)d\theta^2$$

$$R^2(r) = \frac{3(\nu^2-1)}{4}r(r-r_0)$$

$$N^2(r) = \frac{\ell^2(\nu^2+3)(r-r_+)(r-r_-)}{4R(r)^2}$$

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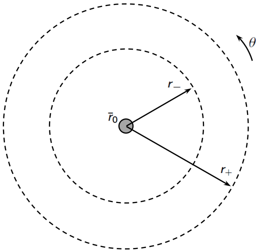

Anninos, Li, Padi, Song, Strominger (2008)

$\nu > 1$ is the **warp factor** of the spacetime.

$\nu \to 1$ recovers the **BTZ black hole**.

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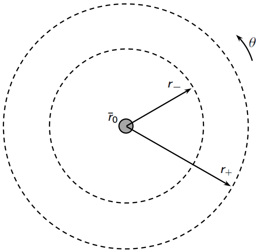

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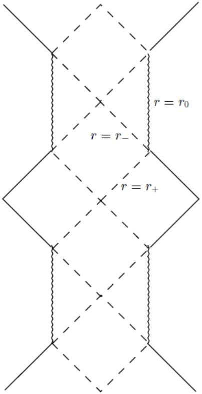
  $\Longrightarrow$ Observers at radius r follow orbits of $d/dt + \Omega(r)\, d/d\theta$!

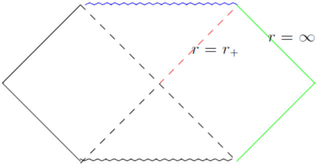
  $\Omega(r)$ is restricted to a range of values and:

$$\Omega(r) \to \Omega_{\mathcal{H}}\,, \quad r \to r_+$$
$$\Omega(r) \to 0\,, \quad r \to +\infty$$

Penrose diagram of a spacelike stretched black hole

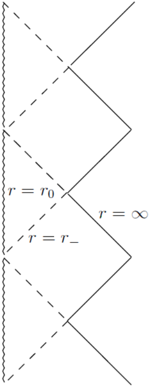


$r_0 < r_- < r_+$



$r_0 = r_- < r_+$

- **Not** asymptotically AdS_3, but similar to asymptotically flat black holes!
- Arena to obtain exact results for a rotating black hole whose causal structure is similar to the Kerr spacetime!



$r_0 < r_- = r_+$

Jugeau, Moutsopoulos, Ritter (2010)

Stable?

So... is it stable or not?

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Today I will consider **classical** stability.

Outline

Scalar field in the background of a $WAdS_3$ black hole

- **Scalar field** Φ in the background of a spacelike stretched black hole:

$$\left(\nabla^2 - m^2\right)\Phi(t, r, \theta) = 0$$

Scalar field in the background of a $WAdS_3$ black hole

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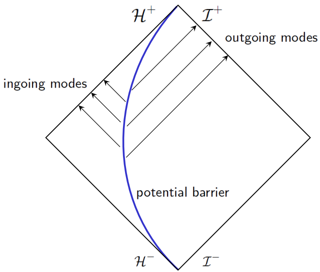
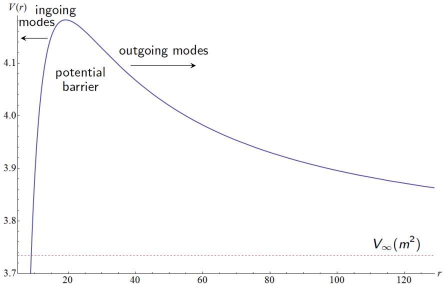
- **Exact** mode solutions:

$$\Phi_{\omega k}(t, r, \theta) \sim e^{-i\omega t + ik\theta} \, z^{\alpha}(1 - z)^{\beta} F(a, b, c; z)$$

  - α, β, a, b, c constants (given in terms of ω and k)
  - $z = (r - r_+)/(r - r_-)$
  - $F(a, b, c; z)$ hypergeometric function

What are quasinormal modes?





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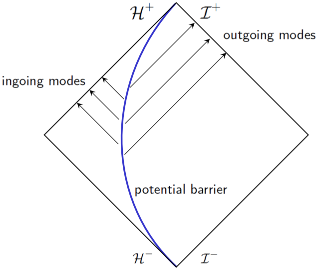

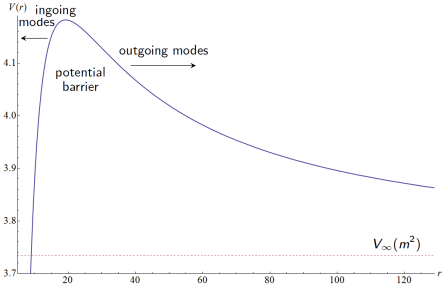

Boundary conditions:

- **Ingoing** modes at the event horizon;
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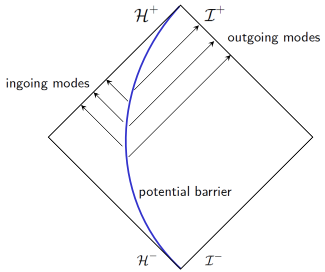

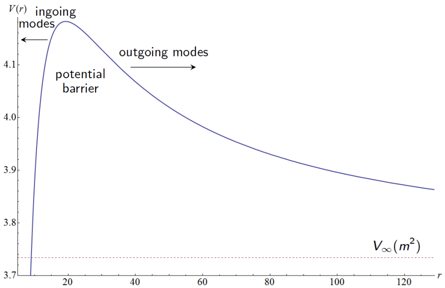

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$\Longrightarrow$ Discrete set of complex eigenfrequencies $\{\omega_n\}$

$$\Phi_n \sim e^{-i\omega_n t} = e^{-i\mathrm{Re}(\omega_n)t + \mathrm{Im}(\omega_n)t}$$

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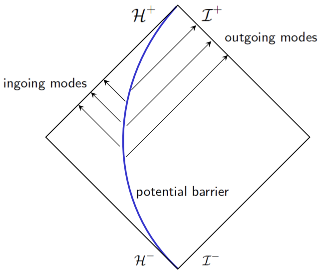

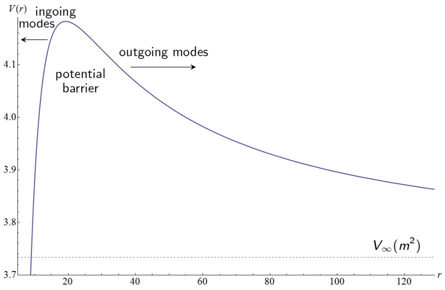

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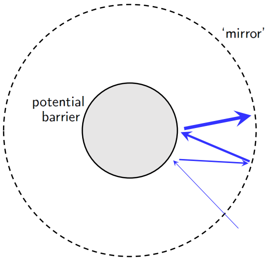
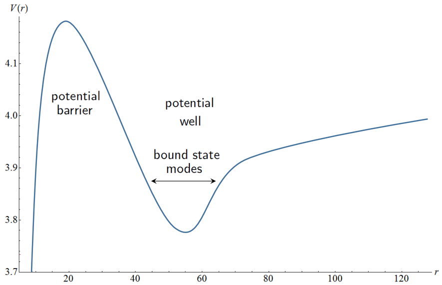
If $\mathrm{Im}(\omega_n) > 0$ for some n: mode is **unstable**!

Superradiant instabilities?





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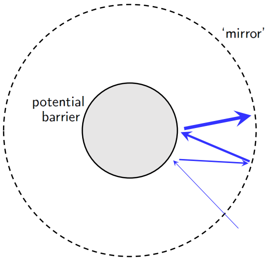

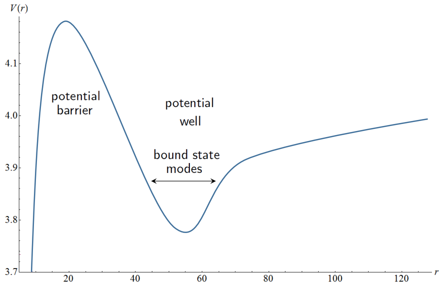

Superradiant modes: amplitude **increases** after reflection if $0 < \mathrm{Re}(\omega) < k\Omega_{\mathcal{H}}$.

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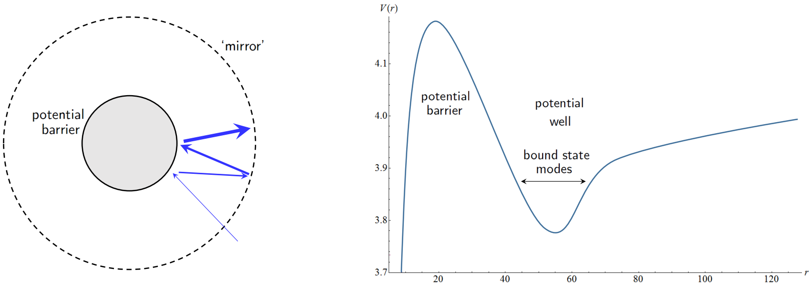

Superradiant modes: amplitude **increases** after reflection if $0 < \mathrm{Re}(\omega) < k\Omega_{\mathcal{H}}$.

Bound state modes: superradiant modes that are localised in the potential well (ingoing at event horizon, exponentially decreasing at infinity).

$\implies \mathrm{Im}(\omega_n) > 0 \implies$ **superradiant instability**!

Are there any unstable modes?

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... NO!

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- All quasinormal modes have $\mathrm{Im}(\omega_n) < 0$.
- **No** bound state modes!
- **No** superradiant modes in warped AdS_3 black holes (proof?).

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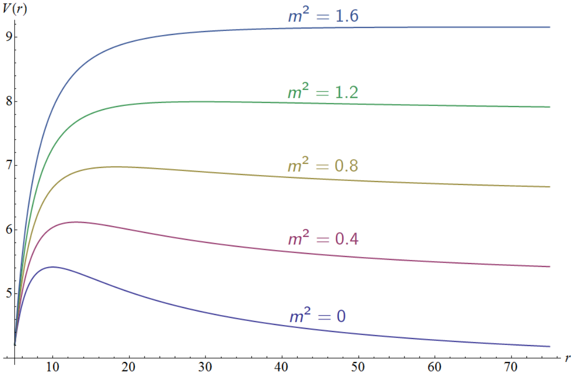

Eigenfrequencies

We can obtain exact expressions for the eigenfrequencies!

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$+$ quasinormal modes $\qquad$ $-$ 'exponentially decreasing modes'

- 'Right eigenfrequencies'

$$(\omega_{\pm})_n^{(R)} = \frac{\nu^2+3}{d^2\delta^2 - 3(\nu^2-1)} \left\{ -d\delta \left(\frac{4kd}{\nu^2+3} + i\left(n+\frac{1}{2}\right)\right) \pm i(e - i\,\mathrm{sgn}(k) f) \right\}$$

$$d = \frac{1}{r_+ - r_-}, \qquad \delta = 2\nu(r_+ + r_-) - 2\sqrt{(\nu^2+3)r_+ r_-},$$

$$e = \sqrt{\frac{\sqrt{E^2+F^2}+E}{2}}, \qquad f = \sqrt{\frac{\sqrt{E^2+F^2}-E}{2}},$$

$$E = \left(\frac{1}{4} + \frac{m^2}{\nu^2+3}\right) d^2\delta^2 - 3(\nu^2-1)\left[\left(\frac{1}{4}+\frac{m^2}{\nu^2+3}\right) + \left(\frac{4kd}{\nu^2+3}\right)^2 - \left(n+\frac{1}{2}\right)^2\right]$$

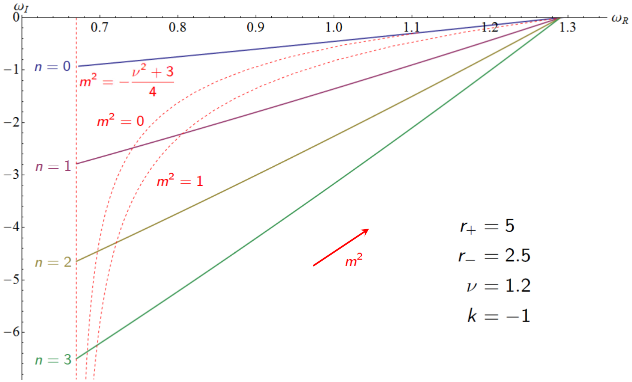
$$F = -3(\nu^2-1)\left(n+\frac{1}{2}\right)\frac{8kd}{\nu^2+3}.$$

- 'Left eigenfrequencies'

$$(\omega_{\pm})_n^{(L)} = -i\left[(2n+1)\nu \mp \sqrt{3(\nu^2-1)\left(n+\frac{1}{2}\right)^2 + (\nu^2+3)\left(\frac{1}{4}+\frac{m^2}{\nu^2+3}\right)}\right]$$

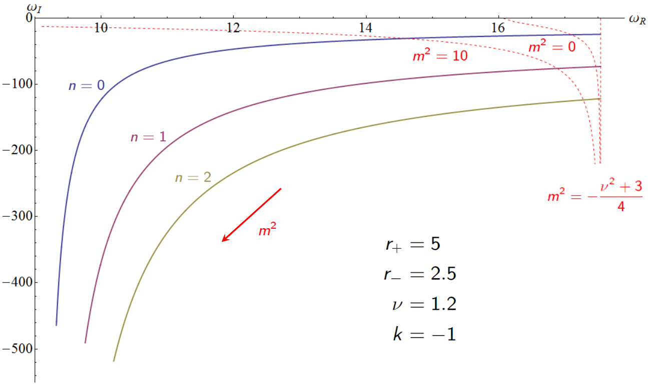
Quasinormal modes

Quasinormal eigenfrequencies in the complex plane w.r.t. m^2



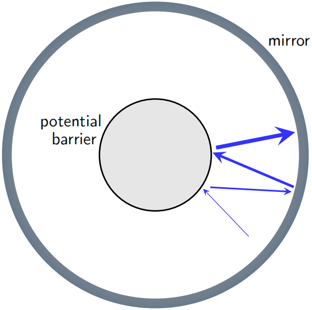
For each n quasinormal modes exist when $m^2 < m^2_{\text{max}}(n)$

'Exponentially decreasing modes' eigenfrequencies w.r.t. m^2



Note: in the context of (W)AdS/CFT, these modes obeying Dirichlet boundary conditions at infinity are *chosen* to be the quasinormal modes.

What happens if we put an actual mirror?



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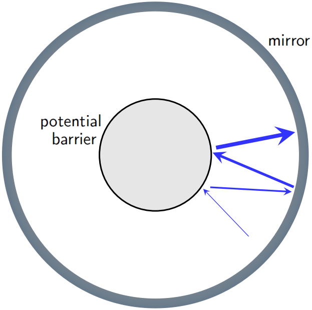

Boundary conditions:

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- **Vanishing** modes at the mirror (Dirichlet boundary condition).

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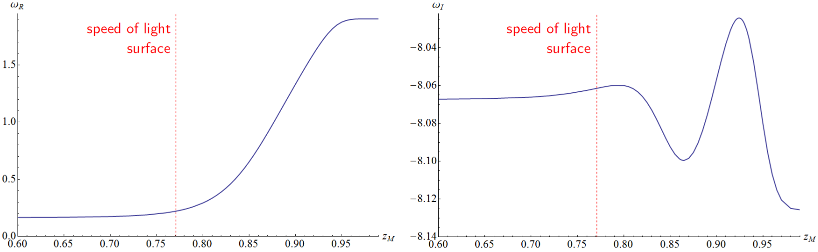
Eigenfrequency vs position of the mirror



$$(r_+ = 7\,, \quad r_- = 0.7\,, \quad \nu = 1.2\,, \quad k = -1\,, \quad m^2 = 0)$$

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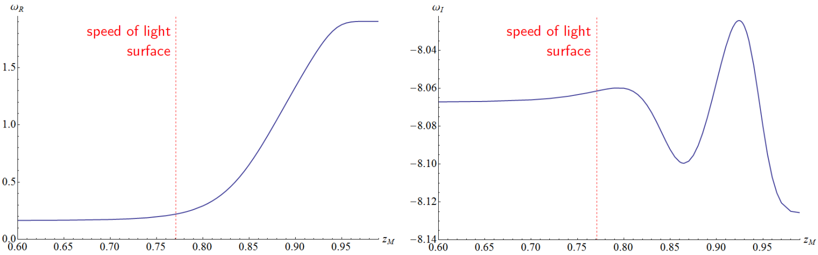

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Eigenmodes are still **stable**!

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Conlusions

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- Warped AdS_3 black holes are interesting (and possibly stable!) solutions of an extension of Einstein gravity in (2+1)-dimensions which has local degrees of freedom.
- Warped AdS_3 black holes has very similar properties to the (3+1)-dimensional Kerr spacetime and, contrary to the latter, many analytical computations can be performed!
- Warped AdS_3 black holes are classically stable to scalar field perturbations, even if a mirror is added to the spacetime.

What's next?

- Is the warped AdS_3 black hole classically stable to other types of pertubations (namely gravitational perturbations)?

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- Is the warped AdS_3 black hole classically stable to other types of pertubations (namely gravitational perturbations)?
- How about quantum effects? Is there a Hartle-Hawking-like vacuum state? What is the renormalised stress-energy tensor for a field in this state?

THANK YOU FOR YOUR ATTENTION!