Vacuum Spacetimes with a constant Weyl scalar

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The Problem

- Vacuum spacetimes with a cosmological constant Λ are considered

$$R_{ij} = \Lambda g_{ij}$$

in which there is an eigen-bivector V^{ij} of the (self-dual) Weyl tensor with a constant eigenvalue λ

$$C^{\dagger ij}{}_{kl} V^{kl} = \lambda V^{ij}$$

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  - OR a special Petrov type I spacetime with zero Weyl scalar; but these do not to exist (Brans, 1975)
  - So the Petrov types to be considered are I, II & D only

Petrov Classification I

There are many approaches to the Petrov classification; all of which are essentially equivalent in 4-D:

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- eigenvector/invariant subspace structure of $Q_{ij} = E_{ij} + iB_{ij}$ where E_{ij} & B_{ij} are the electric and magnetic parts of the Weyl tensor wrt a unit timelike vector u^i

$$E_{ij} = C_{ikjl}u^k u^l \qquad H_{ij} = C^*_{ikjl}u^k u^l$$

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- Number of distinct principal null directions ℓ^i satisfying $\ell_{[m}\ell^j C_{i]jk[l}\ell^k \ell_{n]} = 0$
- Number of distinct 1-spinors in the totally symmetric Weyl spinor $\Psi_{ABCD} = \alpha_{(A}\beta_B\gamma_C\delta_{D)}$

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Petrov Classification II

- For Petrov types I and D there is an orthonormal eigen-tetrad $(u^i, e^i_1, e^i_2, e^i_3)$ of Q_{ij} such that its frame components $(A, B = 1 \ldots 3)$ are

$$Q_{AB} = \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix} \qquad \lambda_1 + \lambda_2 + \lambda_3 = 0$$

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- For Petrov types II the corresponding frame components are

$$Q_{AB} = \begin{pmatrix} 1 - \lambda_3/2 & -i & 0 \\ -i & 1 - \lambda_3/2 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix}$$

Petrov Classification III

Using the associated null tetrad:

$$\ell^i = 1/\sqrt{2}(u^i + e_3^i), \quad n^i = 1/\sqrt{2}(u^i - e_3^i), \quad m^i = 1/\sqrt{2}(e_1^i + i e_2^i)$$

the Weyl tensor components take the form:

- For Petrov Type I:

$$\Psi_1 = \Psi_3 = 0, \quad \Psi_2 = -\lambda_3/2, \quad \Psi_0 = \Psi_4 = (\lambda_2 - \lambda_1)/2$$

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- For Petrov Type D:

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- For Petrov Type D:

$$\Psi_1 = \Psi_3 = 0, \quad \Psi_2 = -\lambda_3/2, \quad \Psi_0 = \Psi_4 = 0$$

- We will assume that λ_3 (and so Ψ_2) is constant.

Algebraically Special Cases (Type II & D)

In the Newman-Penrose spin coefficient formalism the relevant Bianchi identites are (since $\Psi_0 = \Psi_1 = \Psi_3 = 0$ and $\kappa = \sigma = 0$ by the Goldberg-Sachs theorem):

$$\begin{aligned} D(\Psi_2 + R/12) &= 3\rho\Psi_2 \\ \delta(\Psi_2 + R/12) = &= 3\tau\Psi_2 \\ \bar{\delta}(\Psi_2 + R/12) &= -3\pi\Psi_2 \\ \Delta(\Psi_2 + R/12) &= -3\mu\Psi_2 \end{aligned}$$

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Thus, as $\Psi_2 (\neq 0)$ and $R (= 4\Lambda)$ are constant:

$$\kappa = \sigma = \rho = \tau = 0 \quad (\text{and} \quad \pi = \mu = 0)$$

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Thus, as $\Psi_2 (\neq 0)$ and $R (= 4\Lambda)$ are constant:

$$\kappa = \sigma = \rho = \tau = 0 \quad (\textbf{and} \quad \pi = \mu = 0)$$

In addition, from the NP equation for $\Delta\rho - \bar{\delta}\tau$ or $D\mu - \delta\pi$, it follows that $\Psi_2 = -\Lambda/3$.

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Algebraically Special Cases 2

- Thus the spacetime is a Kundt (1961) spacetime and the metric can be written in the form:

$$ds^2 = 2P^{-2}dzd\bar{z} - 2du(dv + Wdz + \bar{W}d\bar{z} + Hdu)$$

where $P = P(z, \bar{z}, u)$ and $H = H(z, \bar{z}, u, v)$ are real and $W = W(z, \bar{z}, u, v)$ is complex.

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- These spacetimes were studied by Lewandowski (1992). Using the field equations and the coordinate freedom preserving the form of the metric, he showed

$$P = 1 + \Lambda z\bar{z}/2 \quad H = -\Lambda v^2/2 + H_0(z, \bar{z}, u), \quad W = iL_{,z}$$

where L is a real potential satisfying $P^2 L_{,z\bar{z}} = -\Lambda L$.

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Algebraically Special Cases 3

- The general solution for the potential L is

$$L = \Re(\Lambda P^{-1}\bar{z}f(z,u) - f_{,z}(z,u))$$

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- For type D the condition $\Psi_4 = 0$ implies $W = g(\bar{z},u)P^{-2}$ and $H_0 = h(u)$. The remaining coordinate freedom preserving the form of the metric can be used to set $W = h(u) = 0$.

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- For type D the condition $\Psi_4 = 0$ implies $W = g(\bar{z},u)P^{-2}$ and $H_0 = h(u)$. The remaining coordinate freedom preserving the form of the metric can be used to set $W = h(u) = 0$. The general type D metric is decomposable into 2 2-spaces of constant curvature:

$$ds^2 = 2(1 + \Lambda z\bar{z}/2)^{-2} dz d\bar{z} - 2du dv - \Lambda v^2 du^2$$

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Algebraically General Case (Petrov type I)

- For simplicity in this talk I consider only the case where all three Weyl scalars are constant. Thus Ψ_2 and $\Psi_0 (= \Psi_4)$ are both constants.

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- The Bianchi identities reduce in this case to purely algebraic equations:

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\begin{aligned}
(\pi - 4\alpha)\Psi_0 &= 3\kappa\Psi_2 & \qquad (\mu - 4\gamma)\Psi_0 &= 3\sigma\Psi_2 \\
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$$

Thus, writing $\psi = \Psi_0/(3\Psi_2)$, we have

$$\rho = \psi\lambda \quad \tau = \psi\nu \quad \mu = \psi\sigma \quad \pi = \psi\kappa, \quad \alpha = (\psi + \psi^{-1})\kappa/4,$$
$$\beta = (\psi + \psi^{-1})\nu/4, \quad \epsilon = (\psi + \psi^{-1})\lambda/4, \quad \gamma = (\psi + \psi^{-1})\sigma/4.$$

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- These are only consistent if κ, σ, ν & λ are all constants. Also $\Lambda = 0$ (pure vacuum only) and $\psi = 1/\sqrt{3}$.
- The last equation means that the 3 Weyl scalars are a constant real multiple of the three cube roots of -1.

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- Note also that the type D metric derived earlier is also homogeneous although the group of isometries is multiply transitive.

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- This metric is originally due to Petrov (1962) who derived it using group theoretic methods.
- Note also that the type D metric derived earlier is also homogeneous although the group of isometries is multiply transitive.
- We have *a fortiori* proved that there are no type I homogeneous vacuum spacetimes with $\Lambda \neq 0$.

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- Coley et al. (2006) proved an analogous result: any spacetime for which all invariants constructed from the curvature tensor and its derivatives of all orders are constant is either homogeneous or Type II.

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- Coley et al. (2006) proved an analogous result: any spacetime for which all invariants constructed from the curvature tensor and its derivatives of all orders are constant is either homogeneous or Type II. In this work we assume the vacuum condition (with Λ) and the constancy of only the algebraic invariants of the curvature tensor.